



Solutions d'équations différentielles

Forme, premier ordre	Solution, condition initiale $f(0) = A$	Remarque
$\frac{d}{dx} f = 0$	$f = A$	
$\frac{d}{dx} f + a = 0$	$f = A - ax$	
$\frac{d}{dx} f + ax + b = 0$	$f = A - bx - \frac{1}{2}ax^2$	
$\frac{d}{dx} f + ax^2 + bx + c = 0$	$f = A - cx - \frac{1}{2}bx^2 - \frac{1}{3}ax^3$	
$\frac{d}{dx} f + a \cdot e^{bx} + c = 0$	$f = A - cx - \frac{a}{b} \cdot (e^{bx} - 1)$	
$\frac{d}{dx} f + af = 0$	$f = A \cdot e^{-ax}$	Asymptote : $\alpha = 0$
$\frac{d}{dx} f + af + b = 0$	$f = \left(A + \frac{b}{a} \right) \cdot e^{-ax} - \frac{b}{a}$	Asymptote : $\alpha = -\frac{b}{a}$
$\frac{d}{dx} f + af + bx + c = 0$	$f = \left(A - \frac{b}{a^2} + \frac{c}{a} \right) \cdot e^{-ax} - \frac{b}{a} \cdot x + \left(\frac{b}{a^2} - \frac{c}{a} \right)$	Asymptote : $\alpha = -\frac{b}{a} \cdot x + \frac{b}{a^2} - \frac{c}{a}$
$\frac{d}{dx} f + af + bx^2 + cx + d = 0$	$f = \left(A + \frac{2b}{a^3} - \frac{c}{a^2} + \frac{d}{a} \right) \cdot e^{-ax} - \frac{b}{a} \cdot x^2 + \left(\frac{2b}{a^2} - \frac{c}{a} \right) \cdot x - \left(\frac{2b}{a^3} - \frac{c}{a^2} + \frac{d}{a} \right)$	
$\frac{d}{dx} f + af + b \cdot e^{cx} + d = 0$	$f = \left(A + \frac{b}{a+c} + \frac{d}{a} \right) \cdot e^{-ax} - \frac{b}{a+c} \cdot e^{cx} - \frac{d}{a}$	Asymptote : $\alpha = -\frac{d}{a}$
$\frac{d}{dx} f + axf = 0$	$f = A \cdot e^{-\frac{1}{2}ax^2}$	Asymptote : $\alpha = 0$

$\frac{d}{dx} f + axf + bx = 0$	$f = \left(A + \frac{b}{a} \right) \cdot e^{-\frac{1}{2}ax^2} - \frac{b}{a}$	Asymptote : $\alpha = -\frac{b}{a}$
$\frac{d}{dx} f + axf + bf = 0$	$f = A \cdot e^{-\frac{1}{2}ax^2 - bx}$	Asymptote : $\alpha = 0$
$\frac{d}{dx} f + af^2 + bf = 0$	$f = \frac{1}{\left(\frac{1}{A} + \frac{a}{b} \right) \cdot e^{bx} - \frac{a}{b}}$ ou $f = \frac{1}{\frac{-a^2 A}{abA + b^2} \cdot e^{-bx} + \frac{a}{b}} - \frac{b}{a}$	Asymptote : $\alpha = 0$ si $b > 0$
$\frac{d}{dx} f + af^2 + bf + c = 0$	$f = \frac{1}{\frac{a}{\sqrt{\Delta}} \cdot \left(\left(\frac{2aA + b + \sqrt{\Delta}}{2aA + b - \sqrt{\Delta}} \right) \cdot e^{\sqrt{\Delta} \cdot x} - 1 \right)} + \frac{-b + \sqrt{\Delta}}{2a}$	$\Delta = b^2 - 4ac$ La solution est identique lorsque $\sqrt{\Delta}$ est remplacé par $-\sqrt{\Delta}$

Forme, deuxième ordre	Solution, conditions initiales $f(0) = A$; $f'(0) = B$	Remarque
$\frac{d^2}{dx^2} f = 0$	$f = A + Bx$	
$\frac{d^2}{dx^2} f + a = 0$	$f = A + Bx - \frac{1}{2}ax^2$	
$\frac{d^2}{dx^2} f + ax + b = 0$	$f = A + Bx - \frac{1}{2}bx^2 - \frac{1}{6}ax^3$	
$\frac{d^2}{dx^2} f + ax^2 + bx + c = 0$	$f = A + Bx - \frac{1}{2}cx^2 - \frac{1}{6}bx^3 - \frac{1}{12}ax^4$	
$\frac{d^2}{dx^2} f + a \cdot e^{bx} + c = 0$	$f = \left(A + \frac{a}{b^2} \right) + \left(B + \frac{a}{b} \right) \cdot x - \frac{1}{2}cx^2 - \frac{a}{b^2} \cdot e^{bx}$	

Les équations linéaires du 2^{ème} ordre à coefficient constant $\frac{d^2}{dx^2} f + a \cdot \frac{d}{dx} f + b \cdot f + c = 0$ se calculent à l'aide de l'équation

caractéristique $r^2 + ar + b = 0$. Les solutions sont : $r_1 = \frac{a}{2} + \frac{\sqrt{a^2 - 4b}}{2}$ et $r_2 = \frac{a}{2} - \frac{\sqrt{a^2 - 4b}}{2}$.

Forme, deuxième ordre	Solution, conditions initiales $f(0) = A ; f'(0) = B$	Remarque
$\frac{d^2}{dx^2} f - a^2 f = 0$	$f = \left(\frac{A}{2} + \frac{B}{2a}\right) \cdot e^{ax} + \left(\frac{A}{2} - \frac{B}{2a}\right) \cdot e^{-ax}$	Solution caract. $r_1 = a$ et $r_2 = -a$
$\frac{d^2}{dx^2} f - a^2 f + b = 0$	$f = \left(\frac{A}{2} + \frac{B}{2a} - \frac{b}{2a^2}\right) \cdot e^{ax} + \left(\frac{A}{2} - \frac{B}{2a} - \frac{b}{2a^2}\right) \cdot e^{-ax} + \frac{b}{a^2}$	Solution caract. $r_1 = a$ et $r_2 = -a$
$\frac{d^2}{dx^2} f + a^2 f = 0$	$f = \frac{B}{a} \cdot \sin(ax) + A \cdot \cos(ax)$	Solutions caract. $r_1 = ia$ et $r_2 = -ia$
$\frac{d^2}{dx^2} f + a^2 f + b = 0$	$f = \frac{B}{a} \cdot \sin(ax) + \left(A + \frac{b}{a^2}\right) \cdot \cos(ax) - \frac{b}{a^2}$	Solutions caract. $r_1 = ia$ et $r_2 = -ia$
$\frac{d^2}{dx^2} f + a \cdot \frac{d}{dx} f = 0$	$f = -\frac{B}{a} \cdot e^{-ax} + A + \frac{B}{a}$	Solutions caract. $r_1 = 0$ et $r_2 = -a$ Asymptote : $\alpha = A + \frac{B}{a}$
$\frac{d^2}{dx^2} f + a \cdot \frac{d}{dx} f + b = 0$	$f = \left(-\frac{b}{a^2} - \frac{B}{a}\right) \cdot e^{-ax} - \frac{b}{a} \cdot x + \left(A + \frac{B}{a} + \frac{b}{a^2}\right)$	Solutions caract. $r_1 = 0$ et $r_2 = -a$ Asymptote : $\alpha = -\frac{b}{a} \cdot x + \left(A + \frac{B}{a} + \frac{b}{a^2}\right)$
$\frac{d^2}{dx^2} f + a \cdot \frac{d}{dx} f + \frac{a^2}{4} \cdot f = 0$	$f = \left(\left(B + \frac{aA}{2}\right) \cdot x + A\right) \cdot e^{-\frac{1}{2}ax}$	Solution caract. $r = -\frac{a}{2}$ Asymptote : $\alpha = 0$
$\frac{d^2}{dx^2} f + a \cdot \frac{d}{dx} f + \frac{a^2}{4} \cdot f + b = 0$	$f = \left(\left(B + \frac{aA}{2} + \frac{2b}{a}\right) \cdot x + \left(A + \frac{4b}{a^2}\right)\right) \cdot e^{-\frac{1}{2}ax} - \frac{4b}{a^2}$	Solution caract. $r = -\frac{a}{2}$ Asymptote : $\alpha = -\frac{4b}{a^2}$
$\frac{d^2}{dx^2} f + a \cdot \frac{d}{dx} f + bf = 0$ Si $a^2 - 4b > 0$	$f = \left(\frac{aA + 2B}{2\sqrt{a^2 - 4b}} + \frac{A}{2}\right) \cdot e^{\frac{-a + \sqrt{a^2 - 4b}}{2} \cdot x} + \left(-\frac{aA + 2B}{2\sqrt{a^2 - 4b}} + \frac{A}{2}\right) \cdot e^{\frac{-a - \sqrt{a^2 - 4b}}{2} \cdot x}$	Solutions caract. $r = -\frac{a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2}$ Asymptote : $\alpha = 0$
$\frac{d^2}{dx^2} f + a \cdot \frac{d}{dx} f + bf = 0$ Si $a^2 - 4b < 0$	$f = \left(\frac{aA + 2B}{\sqrt{4b - a^2}} \cdot \sin\left(\frac{\sqrt{4b - a^2}}{2} \cdot x\right) + A \cdot \cos\left(\frac{\sqrt{4b - a^2}}{2} \cdot x\right)\right) \cdot e^{\frac{1}{2}ax}$	Solutions caract. $r = -\frac{a}{2} \pm i \cdot \frac{\sqrt{4b - a^2}}{2}$ Asymptote : $\alpha = 0$

$\frac{d^2}{dx^2} f + a \cdot \frac{d}{dx} f + bf + c = 0$ <p>Si $a^2 - 4b > 0$</p>	$f = \left(\frac{aA + 2B + \frac{ac}{b}}{2\sqrt{a^2 - 4b}} + \frac{A}{2} + \frac{c}{2b} \right) \cdot e^{\frac{-a + \sqrt{a^2 - 4b}}{2} \cdot x} + \left(-\frac{aA + 2B + \frac{ac}{b}}{2\sqrt{a^2 - 4b}} + \frac{A}{2} + \frac{c}{2b} \right) \cdot e^{\frac{-a - \sqrt{a^2 - 4b}}{2} \cdot x} - \frac{c}{b}$	<p>Solutions caract. $r = -\frac{a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2}$</p> <p>Asymptote : $\alpha = -\frac{c}{b}$</p>
$\frac{d^2}{dx^2} f + a \cdot \frac{d}{dx} f + bf + c = 0$ <p>Si $a^2 - 4b < 0$</p>	$f = \left(\frac{aA + 2B + \frac{ac}{b}}{\sqrt{4b - a^2}} \cdot \sin\left(\frac{\sqrt{4b - a^2}}{2} \cdot x\right) + \left(A + \frac{c}{b}\right) \cdot \cos\left(\frac{\sqrt{4b - a^2}}{2} \cdot x\right) \right) \cdot e^{\frac{-1}{2}ax} - \frac{c}{b}$	<p>Solutions caract. $r = -\frac{a}{2} \pm i \cdot \frac{\sqrt{4b - a^2}}{2}$</p> <p>Asymptote : $\alpha = -\frac{c}{b}$</p>